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Bergshoeff, Eric; Sezgin, Ergin; Proeyen, Antoine Van

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(2, 0) tensor multiplets and conformal supergravity in $D = 6$ Eric Bergshoeff[†], Ergin Sezgin[‡] and Antoine Van Proeyen[§][†] Institute for Theoretical Physics, Nijenborgh 4, 9747 AG Groningen, The Netherlands[‡] Center for Theoretical Physics, Texas A&M University, College Station, TX 77843–4242, USA[§] Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium

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Abstract. We construct the supercurrent multiplet that contains the energy–momentum tensor of the (2, 0) tensor multiplet. By coupling this multiplet of currents to the fields of conformal supergravity, we first construct the linearized superconformal transformations rules of the (2, 0) Weyl multiplet. Next, we construct the full nonlinear transformation rules by gauging the superconformal algebra $OSp(8^*|4)$. We then use this result to construct the full equations of motion of the tensor multiplet in a conformal supergravity background. Coupling $N + 5$ copies of the tensor multiplet to conformal supergravity and imposing a geometrical constraint on the scalar fields which fixes the conformal symmetry, we obtain the coupling of (2, 0) Poincaré supergravity to N tensor multiplets in which the physical scalars parametrize the coset $SO(N, 5)/(SO(N) \times SO(5))$.

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1. Introduction

There is increasing evidence for the fact that M -theory on anti de Sitter (AdS) backgrounds can be described by a conformal field theory at the boundary of AdS, at least in a suitable limit. The low-energy limits involved in the bulk of AdS are the gauged supergravity theories in various dimensions and the boundary field theories are certain globally supersymmetric field theories appropriate to the branes involved.

In verifying the AdS/CFT correspondence, the boundary values of the bulk fields naturally arise. In accordance with the fact that AdS supersymmetry in a given dimension acts as the conformal supersymmetry at the boundary of AdS, the boundary values of the bulk fields are in one-to-one correspondence with the fields of conformal supergravity defined at the boundary. Thus, it is natural to formulate the boundary field theory in a conformal supergravity background. Integration over the boundary (matter) fields should then yield an effective action involving the conformal supergravity fields, which is to be compared with the bulk supergravity effective action.

This approach was followed in [1], where the coupling of $N = 4$, $D = 4$ super Yang–Mills to $N = 4$, $D = 4$ conformal supergravity [2] was studied, as the boundary field theory associated with gauged supergravity in AdS_5 . In this spirit, we wish to construct the coupling of the (2, 0) tensor multiplet to (2, 0) conformal supergravity at the six-dimensional boundary of AdS_7 . This result is expected to provide a convenient framework in studying the AdS_7/CFT_6 correspondence. Since the (2, 0) tensor multiplet contains a chiral 2-form, it is natural to

|| Onderzoeksdirecteur FWO, Belgium.

study its field equations rather than an action from which they may be derivable but which may require the introduction of additional fields. Thus, we shall primarily study the covariant field equations, although we shall briefly discuss an action from which all but the self-duality condition follows, provided that the self-duality equation is imposed after the variation.

The conformal supergravity fields form an off-shell multiplet. We can treat them as background fields, in which case we need not impose their equations of motion. However, coupling of conformal supergravity to $N + 5$ copies of the $(2, 0)$ tensor multiplet, we can constrain the scalars fields coming from these tensor multiplets so that they become a representative of the coset $\frac{SO(N,5)}{SO(N) \times SO(5)}$. As we will show, this leads to a conformal interpretation of the $(2, 0)$ Poincaré supergravity coupled to N tensor multiplets constructed previously in [3, 4].

The organization of this paper is as follows. In section 2, we derive the multiplet of supercurrents and the linearized Weyl multiplet. In section 3, we construct the full $(2, 0)$ conformal supergravity theory. As we did before for the $(1, 0)$ case [5], we follow the methods developed first for $N = 1$ in four dimensions [6]. They are based on gauging the conformal superalgebra [7], which, in our case, is $OSp(8^*|4)$. As is typical in this method, one then has to impose constraints on some of the curvatures. In section 4, we find the complete equations of motions for a single tensor multiplet in a conformal supergravity background. In section 5, we consider $N + 5$ copies of the tensor multiplet in conformal supergravity background and show that a geometrical constraint on the scalars leads to the equations of motion of Poincaré supergravity coupled to N tensor multiplets. Further comments on our results and open problems are collected in the conclusions. Our notations and conventions are presented in appendix A and the truncation of our results to the $(1, 0)$ case [5] are described in appendix B, as we used that correspondence to obtain our present results. A superspace description of the tensor, current and Weyl multiplets is given in appendix C.

2. The $(2, 0)$ supercurrent multiplet

In this section we will construct the $(2, 0)$ supercurrent multiplet. Using the invariance of the bilinear couplings between the currents and the corresponding fields we will derive the *linearized* transformation rules of the $(2, 0)$ conformal supergravity multiplet. In the next section we will extend this to the nonlinear case.

Our starting point is the $(2, 0)$ tensor multiplet in $D = 6$ Minkowski spacetime describing $8 + 8$ degrees of freedom[†]. This is the only on-shell $(2, 0)$ matter multiplet in $D = 6$. Its field components are given in table 2. It contains a 2-form potential $B_{\mu\nu}$ whose self-dual field strength is defined by

$$H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]}, \quad H_{\mu\nu\rho} = \frac{1}{3!}\epsilon_{\mu\nu\rho\sigma\lambda\tau}H^{\sigma\lambda\tau} \equiv \tilde{H}_{\mu\nu\rho}. \quad (2.1)$$

It also contains five real scalars ϕ^{ij} ($i = 1, \dots, 4$) which transforms as a 5-plet of $USp(4)$ and a symplectic Majorana–Weyl spinor ψ^i . The basic properties of these fields are tabulated in table 1.

The complete rigid superconformal transformations have been given in [8] where the tensor multiplet was studied as the $M5$ -brane worldvolume supermultiplet. For our present

[†] In this section the tensor multiplet will only play an auxiliary role as a means to construct the supercurrent multiplet. Later, in section 4, the $(2, 0)$ tensor multiplet will be introduced as matter multiplets to be coupled to the conformal supergravity theory constructed in section 3.

Table 1. Fields of the (2, 0) tensor multiplet. We have indicated the various algebraic restrictions on the fields, their $USp(4)$ representations assignments and the Weyl weights w .

Field	Type	Restrictions	$USp(4)$	w
$B_{\mu\nu}$	Boson	Real antisymmetric tensor gauge field	1	0
ψ^i	Fermion	$\gamma_7 \psi^i = -\psi^i$	4	$\frac{5}{2}$
ϕ^{ij}	Boson	$\phi^{ij} = -\phi^{ji} \quad \Omega_{ij} \phi^{ij} = 0$	5	2

purposes we only need the linearized rigid Q -supersymmetry transformation rules

$$\begin{aligned} \delta B_{\mu\nu} &= -\bar{\epsilon} \gamma_{\mu\nu} \psi, \\ \delta \psi^i &= \frac{1}{48} H_{\mu\nu\rho}^+ \gamma^{\mu\nu\rho} \epsilon^i + \frac{1}{4} \not{\partial} \phi^{ij} \epsilon_j, \\ \delta \phi^{ij} &= -4\bar{\epsilon}^{[i} \psi^{j]} - \Omega^{ij} \bar{\epsilon} \psi. \end{aligned} \quad (2.2)$$

The self-dual part of the curvature H_{abc}^+ transforms as

$$\delta H_{abc}^+ = -\frac{1}{2} \bar{\epsilon} \not{\partial} \gamma_{abc} \psi. \quad (2.3)$$

The supersymmetry transformations close provided that the following linearized field equations are satisfied:

$$H^- = \not{\partial} \psi^i = \partial^\mu \partial_\mu \phi^{ij} = 0. \quad (2.4)$$

To construct the current multiplet, we start from the Noether currents of the (2, 0) tensor multiplet. These Noether currents are: the energy-momentum tensor $\theta_{\mu\nu}$, the supersymmetry currents $J_{\mu i}$ and the $USp(4)$ currents v_μ^{ij} . We will use the improved currents that satisfy the following equations:

$$\begin{aligned} \partial^\mu \theta_{\mu\nu} &= 0, & \theta_{\mu\nu} &= \theta_{\nu\mu}, & \theta^\mu{}_\mu &= 0, \\ \partial^\mu J_{\mu i} &= 0, & \gamma^\mu J_{\mu i} &= 0, \\ \partial^\mu v_\mu^{ij} &= 0. \end{aligned} \quad (2.5)$$

These symmetry properties determine the currents up to constants, which we have determined by requiring the closure of the rigid linearized supersymmetry transformations. We thus find the currents

$$\begin{aligned} \theta_{\mu\nu} &= H_{\mu ab}^+ H_{\nu}^{+ab} + 8\bar{\psi} \gamma_{(\mu} \partial_{\nu)} \psi + \partial_\mu \phi^{ij} \partial_\nu \phi_{ij} - \frac{1}{10} \eta_{\mu\nu} (\partial\phi)^2 - \frac{1}{5} \partial_\mu \partial_\nu \phi^2, \\ J_{\mu i} &= -\frac{2}{3} H_{\rho\sigma\tau}^+ \gamma^{\rho\sigma\tau} \gamma_\mu \psi_i + 8\phi_{ij} \overset{\leftrightarrow}{\partial}_\mu \psi^j + \frac{8}{5} \gamma_{\mu\lambda} \partial^\lambda (\phi_{ij} \psi^j), \\ v_\mu^{ij} &= -2\phi_k^{(i} \partial_\mu \phi^{j)k} + 8\bar{\psi}^i \gamma_\mu \psi^j, \end{aligned} \quad (2.6)$$

where $\phi^2 \equiv \phi^{ij} \phi_{ij}$. The currents are conserved provided that the fields satisfy the free field equations (equation (2.4)). When we apply supersymmetry transformations (2.2) on the currents (2.6), always using the field equations (2.4), we find a full supermultiplet of operators bilinear in the fields[†]:

$$\begin{aligned} \delta \theta_{\mu\nu} &= \frac{1}{4} \bar{\epsilon} \gamma_{\rho(\mu} \partial^\rho J_{\nu)}, \\ \delta J_\mu^i &= \gamma^\nu \theta_{\mu\nu} \epsilon^i - \frac{1}{8} (\gamma^\rho \gamma_{\mu\nu} - \frac{3}{5} \gamma_{\mu\nu} \gamma^\rho) \partial^\nu v_\rho^{ij} \epsilon_j - \frac{1}{4} (\gamma^{abc} \gamma_{\mu\nu} + \frac{1}{5} \gamma_{\mu\nu} \gamma^{abc}) \partial^\nu t_{abc}^{ij} \epsilon_j, \\ \delta v_\mu^{ij} &= -\frac{1}{2} \bar{\epsilon}^{(i} J_\mu^{j)} + \frac{15}{8} \bar{\epsilon}_k \gamma_{\mu\nu} \partial^\nu \lambda^{k(i,j)}, \\ \delta t_{abc}^{ij} &= -\frac{1}{24} \bar{\epsilon}^{[i} \gamma^\rho \gamma_{abc} J_{\rho}^{j]} + \frac{5}{32} \bar{\epsilon}^k \not{\partial} \gamma_{abc} \lambda_k^{ij} - (\text{trace}), \\ \delta \lambda_k^{ij} &= -\frac{1}{15} \gamma^{abc} t_{abc}^{ij} \epsilon_k - \frac{4}{15} \gamma^\mu v_{\mu k}^{[i} \epsilon^{j]} - 4 \not{\partial} d_{kl}^{ij} \epsilon^l - (\text{trace}), \\ \delta d^{ij,kl} &= -\frac{1}{8} \bar{\epsilon}^{[i} \lambda^{j],kl} + (ij \leftrightarrow kl) - (\text{trace}), \end{aligned} \quad (2.7)$$

[†] The same operators have been given in [9] using the superfield approach.

Table 2. Fields of (2, 0) conformal supergravity: We have indicated the various algebraic restrictions on the fields, their $USp(4)$ representation assignments, and the Weyl weights w . A field ϕ of weight w transforms under dilatations as $\delta_D \phi = w \Lambda_D \phi$.

Field	Type	Restrictions	$USp(4)$	w
e_μ^a	Boson	Sechsbein	1	-1
ψ_μ^i	Fermion	$\gamma \gamma \psi_\mu^i = +\psi_\mu^i$	4	$-\frac{1}{2}$
V_μ^{ij}	Boson	$V_\mu^{ij} = V_\mu^{ji}$	10	0
T_{abc}^{ij}	Boson	$T_{abc}^{ij} = -T_{abc}^{ji}$ $\Omega_{ij} T_{abc}^{ij} = 0$ $T_{abc}^{ij} = -\frac{1}{6} \epsilon_{abcdef} T_{def}^{ij}$	5	1
χ_k^{ij}	Fermion	$\chi_k^{ij} = -\chi_k^{ji}$ $\Omega_{ij} \chi_k^{ij} = 0$ $\chi_i^{ij} = 0$ $\gamma \gamma \chi_k^{ij} = +\chi_k^{ij}$	16	$\frac{3}{2}$
$D^{ij,kl}$	Boson	$D^{ij,kl} = -D^{ji,kl} = -D^{ij,\ell k}$ $D^{ij,kl} = D^{kl,ij}$ $\Omega_{ij} D^{ij,kl} = \Omega_{kl} D^{ij,kl} = 0$ $\Omega_{ik} \Omega_{j\ell} D^{ij,kl} = 0$	14	2

where we have introduced the operators λ_k^{ij} , t_{abc}^{ij} and $d^{ij,kl}$ defined by

$$\begin{aligned} t_{abc}^{ij} &= \frac{2}{3} H_{abc}^+ \phi^{ij} - \frac{4}{3} \bar{\psi}^i \gamma_{abc} \psi^j - \frac{1}{3} \Omega^{ij} \bar{\psi} \gamma_{abc} \psi, \\ \lambda_k^{ij} &= -\frac{32}{15} \phi^{ij} \psi_k - \frac{128}{75} \delta_k^{[i} \phi^{j]\ell} \psi_\ell + \frac{32}{75} \Omega^{ij} \phi_{k\ell} \psi^\ell, \\ d_{k\ell}^{ij} &= -\frac{1}{15} \phi^{ij} \phi_{k\ell} + \frac{1}{75} \delta_{[k}^{[i} \phi_{\ell]}^{j]} \phi^2 - \frac{1}{300} \Omega^{ij} \Omega_{k\ell} \phi^2. \end{aligned} \quad (2.8)$$

Note that the operator t_{abc}^{ij} is *self-dual*. To prove the transformation rules we have used several $USp(4)$ Schouten identities such as

$$\begin{aligned} 2\bar{\epsilon}^{[i} \phi^{j]k} \psi_k + 2\bar{\epsilon}^k \phi_k^{[i} \psi^{j]} + \phi^{ij} \bar{\epsilon} \psi - (\text{trace}) &= 0, \\ \gamma^{abc} \epsilon_k \bar{\psi}^i \gamma_{abc} \psi^j - 2\gamma^{abc} \epsilon^{[j} \bar{\psi}^{i]} \gamma_{abc} \psi_k - (\text{traces}) &= 0. \end{aligned} \quad (2.9)$$

The currents (2.6) and the operators (2.8) constitute the multiplet of currents. Taking into account the conditions (2.5) this supermultiplet contains 128 + 128 degrees of freedom. Using this current multiplet, the linearized Weyl multiplet is derived by introducing the Noether coupling

$$\int d^6x [h_{\mu\nu} \theta^{\mu\nu} + \bar{\psi}^\mu J_\mu + V_\mu^{ij} v_{ij}^\mu + T_{abc}^{ij} t_{ij}^{abc} + \bar{\chi}_k^{ij} \lambda_{ij}^k + D^{ij,kl} d_{ij,kl}]. \quad (2.10)$$

The independent conformal supergravity fields which have been introduced above are

$$h_{\mu\nu}, \quad \psi_\mu^i, \quad V_\mu^{ij}, \quad T_{abc}^{ij}, \quad \chi_{jk}^i, \quad D^{ij,kl}. \quad (2.11)$$

Several properties of these gauge and matter fields are summarized in table 2.

Demanding the invariance of the action equation (2.10) under rigid supersymmetry, we find the following linearized Weyl multiplet transformation rules:

$$\begin{aligned} \delta h_{\mu\nu} &= \bar{\epsilon} \gamma_{(\mu} \psi_{\nu)}, \\ \delta \psi_\mu^i &= -\frac{1}{4} (\partial_\rho h_{\mu\sigma}) \gamma^{\rho\sigma} \epsilon^i + \frac{1}{2} V_\mu^{ij} \epsilon_j + \frac{1}{24} T_{abc}^{ij} \gamma^{abc} \gamma_\mu \epsilon_j, \\ \delta V_\mu^{ij} &= \frac{1}{4} \bar{\epsilon}^{(i} (\gamma^{\rho\sigma} \gamma_\mu - \frac{3}{5} \gamma_\mu \gamma^{\rho\sigma}) \psi_{\rho\sigma}^{j)} - \frac{4}{15} \bar{\epsilon}_k \gamma_\mu \chi^{(i,j)k}, \\ \delta T_{abc}^{ij} &= \frac{1}{8} \bar{\epsilon}^{[i} (\gamma^{\mu\nu} \gamma_{abc} + \frac{1}{5} \gamma_{abc} \gamma^{\mu\nu}) \psi_{\mu\nu}^{j]} - \frac{1}{15} \bar{\epsilon}^k \gamma_{abc} \chi_k^{ij} - (\text{trace}), \\ \delta \chi_k^{ij} &= \frac{5}{32} (\partial_\mu T_{abc}^{ij}) \gamma^{abc} \gamma^\mu \epsilon_k - \frac{15}{8} \gamma^{\mu\nu} V_{\mu\nu k}^{[i} \epsilon^{j]} - \frac{1}{4} D_{k\ell}^{ij} \epsilon^\ell - (\text{traces}), \\ \delta D^{ij,kl} &= -2\bar{\epsilon}^{[i} \not{\partial} \chi^{j],kl} - 2\bar{\epsilon}^{[k} \not{\partial} \chi^{\ell],ij} - (\text{trace}), \end{aligned} \quad (2.12)$$

where the supersymmetry parameter is constant, and we have defined

$$\begin{aligned}\psi_{\mu\nu} &= \partial_\mu \psi_\nu - \partial_\nu \psi_\mu, \\ V_{\mu\nu}^{ij} &= \partial_\mu V_\nu^{ij} - \partial_\nu V_\mu^{ij}.\end{aligned}\quad (2.13)$$

In the next section, we generalize the above transformation rules to obtain the full local superconformal transformation rules.

3. The (2, 0) conformal supergravity theory

In this section, we will construct the nonlinear (2, 0) conformal supergravity theory. Our starting point is the linearized conformal multiplet constructed in the previous section. This multiplet contains both gauge fields and matter fields (not to be confused with the tensor multiplet matter fields that will be coupled in section 4). Due to the presence of the matter fields, the nonlinearization cannot be understood as a straightforward gauging of an underlying superconformal algebra. To include the matter fields, one must follow a six-step procedure that has been explained in detail in [5]. In the same reference, this six-step procedure has been applied to construct the (1, 0) conformal supergravity theory. Here we apply the same procedure to construct the (2, 0) theory.

The (2, 0) conformal supergravity is based on the superconformal algebra $OSp(8^*|4)$ whose generators are labelled

$$T_A = P_a, \quad Q_{\alpha i}, \quad U_{ij}, \quad M_{ab}, \quad K_a, \quad S_{\alpha i}, \quad D, \quad (3.1)$$

where a, b, \dots are Lorentz indices, α is a chiral spinor index and $i, j = 1, \dots, 4$ are $USp(4)$ indices. M_{ab} and P_a are the Poincaré generators, K_a is the special conformal transformation, D the dilatation, $Q_{\alpha i}$ and $S_{\alpha i}$ are the supersymmetry and special supersymmetry generators, respectively, which are symplectic Majorana–Weyl spinors, 16 real components in total. Finally, $U^{ij} = U^{ji}$ are the $USp(4)$ generators. For more details on the $OSp(8^*|4)$ algebra and the rigid superconformal transformations, see [8].

The gauge fields corresponding to the above generators are

$$e_\mu^a, \quad \psi_\mu^i, \quad V_\mu^{ij}, \quad \omega_\mu^{ab}, \quad f_\mu^a, \quad \phi_\mu^i, \quad b_\mu. \quad (3.2)$$

However, in the realization (Weyl multiplet) which will gauge the algebra, these fields are not all independent. The independent fields are given in (2.11), where the first three are the gauge fields corresponding to the generators P_a , $Q_{\alpha i}$ and U_{ij} . It is understood that the linearized gravitational field $h_{\mu\nu}$ has been replaced by the sechsbein e_μ^a . The last three are matter fields needed for the realization of the superconformal algebra. The remaining gauge fields are either dependent (ω_μ^{ab} , f_μ^a , ϕ_μ^i) (see below), or can be shifted away (b_μ) using K_a invariance.

The (2, 0) Weyl multiplet describes 128+128 off-shell degrees of freedom. We first present the result and next explain our notation and give our definitions. The bosonic transformations of the independent gauge fields are given by general coordinate transformations and

$$\begin{aligned}\delta e_\mu^a &= -\Lambda_D e_\mu^a - \Lambda^{ab} e_{\mu b}, \\ \delta \psi_\mu^i &= -\frac{1}{2} \Lambda_D \psi_\mu^i + \frac{1}{2} \Lambda_j^i \psi_\mu^j - \frac{1}{4} \Lambda^{ab} \gamma_{ab} \psi_\mu^i, \\ \delta V_\mu^{ij} &= \partial_\mu \Lambda^{ij} + \Lambda^{(i}{}_k V_\mu^{j)k}, \\ \delta b_\mu &= \partial_\mu \Lambda_D - 2e_\mu^a \Lambda_{K a}.\end{aligned}\quad (3.3)$$

where Λ_D , $\Lambda_{K a}$, Λ^{ab} and Λ^{ij} are the parameters of dilatation, special conformal, Lorentz and $USp(4)$ transformations, respectively. The transformation properties of the matter fields, T , χ

and D under dilatations, Lorentz and $USp(4)$ transformations follow from the rules (3.3) and from table 2. All matter fields are inert under the special conformal transformations K .

Following [5, 6] we impose the following curvature constraints[†]

$$\begin{aligned} R_{\mu\nu}{}^a(P) &= 0, \\ R_{\mu\nu}{}^{ab}(M)e^v{}_b + \frac{1}{4}T_{\mu bc}^{ij}T_{ij}^{abc} &= 0, \\ \gamma^\mu R_{\mu\nu}^i(Q) &= 0. \end{aligned} \quad (3.4)$$

The above matter-modified curvatures are defined by

$$\begin{aligned} R_{\mu\nu}{}^a(P) &= 2\partial_{[\mu}e_{\nu]}^a + 2b_{[\mu}e_{\nu]}^a + \underline{2\omega_{[\mu}{}^{ab}e_{\nu]}{}_b} - \frac{1}{2}\bar{\psi}_\mu\gamma^a\psi_\nu, \\ R_{\mu\nu}{}^{ab}(M) &= 2\partial_{[\mu}\omega_{\nu]}{}^{ab} + 2\underline{\omega_{[\mu}{}^{ac}\omega_{\nu]}{}_c}{}^b - 8f_{[\mu}{}^{[a}e_{\nu]}{}^{b]} + \bar{\psi}_{[\mu}\gamma^{ab}\phi_{\nu]} \\ &\quad + \bar{\psi}_{[\mu}\gamma^{[a}R_{\nu]}{}^{b]}(Q) + \frac{1}{2}\bar{\psi}_{[\mu}\gamma_{\nu]}R^{ab}(Q) + \frac{1}{2}\bar{\psi}_{\mu,i}\gamma_c\psi_{\nu,j}T^{abc,ij}, \\ R_{\mu\nu}^i(Q) &= (2\partial_{[\mu}\psi_{\nu]}^i + b_{[\mu}\psi_{\nu]}^i + \frac{1}{2}\omega_{[\mu}{}^{ab}\gamma_{ab}\psi_{\nu]}^i - V_{[\mu j}\psi_{\nu]}^j) \\ &\quad + 2\underline{\gamma_{[\mu}\phi_{\nu]}^i} + \frac{1}{12}T_{abc}^{ij}\gamma^{abc}\gamma_{[\mu}\psi_{\nu]}{}_j. \end{aligned} \quad (3.5)$$

The underlined terms indicate all terms of the form gauge field \times sechsbein. Since the sechsbein is invertible the corresponding gauge fields can be solved for from the constraints (3.4). Explicitly, the constraints (3.4) enable us to solve for the gauge fields $(\omega_\mu{}^{ab}, f_\mu{}^a, \phi_\mu^i)$ as follows:

$$\begin{aligned} \omega_\mu{}^{ab} &= 2e^{v[a}\partial_{[\mu}e_{\nu]}{}^{b]} - e^{\rho[a}e^{b]\sigma}e_\mu{}^c\partial_\rho e_{\sigma c} + 2e_\mu{}^{[a}b^{b]} + \frac{1}{2}\bar{\psi}_\mu\gamma^{[a}\psi^{b]} + \frac{1}{4}\bar{\psi}^a\gamma_\mu\psi^b, \\ f_\mu{}^a &= -\frac{1}{8}R_\mu{}^a(M) + \frac{1}{80}e_\mu{}^a R'(M) + \frac{1}{32}T_{\mu cd}^{ij}T_{ij}^{acd}, \\ \phi_\mu^i &= -\frac{1}{16}(\gamma^{ab}\gamma_\mu - \frac{3}{5}\gamma_\mu\gamma^{ab})R_{ab}{}^i(Q). \end{aligned} \quad (3.6)$$

The notation R' indicates that in the corresponding curvature the underlined term in equation (3.5) has been omitted, and $R_\mu{}^a = e_b{}^a R_{\mu\nu}{}^{ba}$.

We next give the full nonlinear Q - and S -transformations of the $(2, 0)$ Weyl multiplet:

$$\begin{aligned} \delta e_\mu{}^a &= \frac{1}{2}\bar{\epsilon}\gamma^a\psi_\mu, \\ \delta b_\mu &= -\frac{1}{2}\bar{\epsilon}\phi_\mu + \frac{1}{2}\bar{\eta}\psi_\mu, \\ \delta\psi_\mu^i &= \mathcal{D}_\mu\epsilon^i + \frac{1}{24}T_{abc}^{ij}\gamma^{abc}\gamma_\mu\epsilon_j + \gamma_\mu\eta^i, \\ \delta V_\mu^{ij} &= -4\bar{\epsilon}^{(i}\phi_\mu^{j)} - \frac{4}{15}\bar{\epsilon}_k\gamma_\mu\chi^{(i,j)k} - 4\bar{\eta}^{(i}\psi_\mu^{j)}, \\ \delta T_{abc}^{ij} &= \frac{1}{8}\bar{\epsilon}^{[i}\gamma^{de}\gamma_{abc}R_{de}^{j]}(Q) - \frac{1}{15}\bar{\epsilon}^k\gamma_{abc}\chi_k^{ij} - (\text{trace}), \\ \delta\chi_k^{ij} &= \frac{5}{32}(\mathcal{D}_\mu T_{abc}^{ij})\gamma^{abc}\gamma^\mu\epsilon_k - \frac{5}{16}\gamma^{\mu\nu}R_{\mu\nu k}{}^{[i}(V)\epsilon^{j]} - \frac{1}{4}D_{kl}^{ij}\epsilon^l \\ &\quad + \frac{5}{8}T_{abc}^{ij}\gamma^{abc}\eta_k - (\text{traces}), \\ \delta D^{ij,kl} &= -2\bar{\epsilon}^{[i}\mathcal{D}\chi^{j],kl} + 4\bar{\eta}^{[i}\chi^{j],kl} + (ij \leftrightarrow kl) - (\text{trace}). \end{aligned} \quad (3.7)$$

We have used here the following definitions. The covariant derivatives and $USp(4)$ curvature in (3.7) are:

$$\begin{aligned} \mathcal{D}_\mu\epsilon^i &= \partial_\mu\epsilon^i + \frac{1}{2}b_\mu\epsilon^i + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon^i - \frac{1}{2}V_\mu{}^i{}_j\epsilon^j, \\ R_{\mu\nu}{}^{ij}(V) &= 2\partial_{[\mu}V_{\nu]}{}^{ij} + V_{[\mu}{}^{k(i}V_{\nu]}{}^{j)k} + 8\bar{\psi}_{[\mu}{}^{(i}\phi_{\nu]}{}^{j)} + \frac{8}{15}\bar{\psi}_{[\mu,k}\gamma_{\nu]}\chi^{(i,j)k}. \end{aligned} \quad (3.8)$$

The supercovariant derivatives \mathcal{D}_μ of matter fields are defined as the ordinary derivative ∂_μ plus a covariantization term which is always given by minus all the transformation rules of the matter field with the parameter replaced by the corresponding gauge field. For example, the supercovariant derivative of T is given by

$$\begin{aligned} \mathcal{D}_\mu T_{abc}^{ij} &= \partial_\mu T_{abc}^{ij} + 3\omega_{\mu[a}{}^dT_{bc]}^{ij} - b_\mu T_{abc}^{ij} + V_\mu{}^{li}T_{abc}^{j]k} - \frac{1}{8}\bar{\psi}_\mu^{[i}\gamma^{de}\gamma_{abc}R_{de}^{j]}(Q) \\ &\quad + \frac{1}{15}\bar{\psi}_\mu{}^k\gamma_{abc}\chi_k^{ij} - (\text{trace}). \end{aligned} \quad (3.9)$$

[†] Note that in contradistinction to the $(1, 0)$ case, discussed in [5], the matter fields χ_{jk}^i and D are absent in the constraints. See also appendix B.

4. The (2, 0) tensor multiplet in the conformal supergravity background

In this section we couple a (2, 0) tensor multiplet to conformal supergravity. Our starting point will be the linearized transformation rules of the tensor multiplet. The nonlinear rules can then be obtained by imposing the superconformal algebra via an iterative Noether procedure. This procedure has been described in detail for the (1, 0) case in [5]. The same procedure can be applied here. As an alternative, we will derive the same result by the requirement that the (2, 0) nonlinear tensor multiplet should reproduce, upon truncation the (1, 0) nonlinear tensor multiplet of [5]. The details of this truncation are explained in appendix B.

Our starting point is the the linearized equations of motion and the supersymmetry transformations of the (2, 0) tensor multiplet given in section 2. Applying the truncation procedure described in appendix B, we find that the full nonlinear Q - and S -transformations of the (2, 0) tensor multiplet are given by:

$$\begin{aligned}\delta B_{\mu\nu} &= -\bar{\epsilon}\gamma_{\mu\nu}\psi + \bar{\epsilon}^i\gamma_{[\mu}\psi_{\nu]}^j\phi_{ij}, \\ \delta\psi^i &= \frac{1}{48}H_{\mu\nu\rho}^+\gamma^{\mu\nu\rho}\epsilon^i + \frac{1}{4}\mathcal{D}\phi^{ij}\epsilon_j - \phi^{ij}\eta_j, \\ \delta\phi^{ij} &= -4\bar{\epsilon}^{[i}\psi^{j]} - (\text{trace}).\end{aligned}\quad (4.1)$$

The curvature $H_{\mu\nu\rho}$ is defined by

$$H_{\mu\nu\rho} = 3\partial_{[\mu}B_{\nu\rho]} + 3\bar{\psi}_{[\mu}\gamma_{\nu\rho]}\psi - \frac{3}{2}\bar{\psi}_{[\mu}^i\gamma_{\nu}\psi_{\rho]}^j\phi_{ij}. \quad (4.2)$$

It satisfies the Bianchi identity

$$\mathcal{D}_{[a}H_{bcd]} - \frac{3}{2}\bar{\psi}_{[ab}\gamma_{cd]}\psi(Q) = 0. \quad (4.3)$$

The self-dual part of the curvature H_{abc}^+ transforms as

$$\delta H_{abc}^+ = -\frac{1}{2}\bar{\epsilon}\mathcal{D}\gamma_{abc}\psi - 3\bar{\eta}\gamma_{abc}\psi. \quad (4.4)$$

Furthermore, we find that the field equations of the (2, 0) tensor multiplet are given by[†]

$$\begin{aligned}\mathcal{F}_{abc}^- &:= H_{abc}^- - \frac{1}{2}\phi_{ij}T_{abc}^{ij} = 0, \\ \Gamma^i &:= \mathcal{D}\psi^i - \frac{1}{15}\phi^{kl}\chi_{kl}^i - \frac{1}{12}T_{abc}^{ij}\gamma^{abc}\psi_j = 0, \\ C_{ij} &:= \mathcal{D}^a\mathcal{D}_a\phi_{ij} - \frac{1}{15}D_{ij}^{kl}\phi_{kl} + \frac{1}{3}H_{abc}^+T_{ij}^{abc} + \frac{16}{15}\bar{\chi}_{ij}^k\psi_k = 0,\end{aligned}\quad (4.5)$$

where

$$\begin{aligned}\mathcal{D}_\mu\psi^i &= (\partial_\mu - \frac{5}{2}b_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab})\psi^i - \frac{1}{2}V_{\mu j}^i\psi^j - \frac{1}{48}H_{abc}^+\gamma^{abc}\psi_\mu^i - \frac{1}{4}(\mathcal{D}\phi^{ij})\psi_{\mu j} + \phi^{ij}\phi_{\mu j}, \\ \mathcal{D}_\mu\phi^{ij} &= (\partial_\mu - 2b_\mu)\phi^{ij} + V_\mu^{[i}\phi^{j]k} + 4(\bar{\psi}_\mu^{[i}\psi^{j]}) - (\text{trace}).\end{aligned}\quad (4.6)$$

To determine the supercovariant d'Alembertian of the scalars we first calculate the transformation properties of $\mathcal{D}_a\phi^{ij}$:

$$\begin{aligned}\delta\mathcal{D}_a\phi^{ij} &= 3\Lambda_D\mathcal{D}_a\phi^{ij} + \Lambda^{[i}_k\mathcal{D}_a\phi^{j]k} - \Lambda_a{}^b\mathcal{D}_b\phi^{ij} + 4\Lambda_{Ka}\phi^{ij} - 4\bar{\epsilon}^{[i}\mathcal{D}_a\psi^{j]} \\ &\quad + \frac{2}{15}(\bar{\epsilon}_l\gamma_a\chi^{(i,k)l}\phi^{j]}_k - i \leftrightarrow j) + \frac{1}{6}\bar{\epsilon}_k\gamma_a\gamma^{bcd}T_{bcd}^{k[i}\psi^{j]} - 4\bar{\eta}^{[i}\gamma_a\psi^{j]} - (\text{trace}).\end{aligned}\quad (4.7)$$

From this we derive that

$$\begin{aligned}\mathcal{D}^a\mathcal{D}_a\phi^{ij} &= \partial^a\mathcal{D}_a\phi^{ij} - 3b^a\mathcal{D}_a\phi^{ij} + V_a^{[i}\mathcal{D}^a\phi^{j]k} + \omega_a{}^{ab}\mathcal{D}_b\phi^{ij} - 4f_a{}^a\phi^{ij} + 4\bar{\psi}_a^{[i}\mathcal{D}^a\psi^{j]} \\ &\quad - \frac{2}{15}(\bar{\psi}_l^a\gamma_a\chi^{(i,k)l}\phi^{j]}_k - i \leftrightarrow j) - \frac{1}{6}\bar{\psi}_k^a\gamma_a\gamma^{bcd}T_{bcd}^{k[i}\psi^{j]} + 4\bar{\phi}_a^{[i}\gamma^a\psi^{j]} - (\text{trace}).\end{aligned}\quad (4.8)$$

[†] Note that, in contradistinction to the (1, 0) case, the first field equation *cannot* be used to solve for the matter field T in terms of H^- (the scalar ϕ is not a singlet under $USp(4)$). Therefore, the (2, 0) Weyl multiplet has *no* alternative formulation containing an antisymmetric tensor gauge field like the (1, 0) Weyl multiplet (see [5]).

Note the occurrence of the Riemann curvature scalar in the equation of motion for the scalar fields through the term $f_a^a \phi_{ij}$. This contains graviton as well as gravitino terms of the supergravity action, the latter through $f_a^a = -\frac{1}{20}R'(M) + \dots = \frac{1}{160}\bar{\psi}_\mu \gamma^{\mu\nu\rho} R'_{\nu\rho}(Q) + \dots$. Gravitino kinetic terms appear also in the equation of motion for tensor multiplet fermions through the term $\mathcal{D}\psi^i = \phi^{ij}\gamma^\mu \phi_{\mu j} + \dots = \frac{1}{10}\phi^{ij}\gamma^{\mu\nu} R'_{\mu\nu j}(Q) + \dots$.

While it is possible to compute the Green functions for the tensor multiplet fields in the presence of the conformal supergravity background by starting from the equations of motion, it would be convenient to perform such calculations by starting from an action. To construct a manifestly Lorentz-invariant action requires the introduction of an auxiliary scalar field [10]. It has been shown in [8] that this can straightforwardly be implemented in a rigid conformal theory. We expect that this can be extended for the local superconformal case.

An alternative approach is to relax the chirality condition on the 2-form potential and to write an action which is not invariant, but whose variation is proportional to \mathcal{F}_{abc}^- . It gives the correct equations of motion provided that the self-duality condition $\mathcal{F}_{abc}^- = 0$ is imposed after the action is varied [4, 11–13]. Such an action takes the form

$$S = \int d^6x \left(-\frac{1}{6} H^{abc} \mathcal{F}_{abc}^- - 4\bar{\psi}^i \Gamma_i - \bar{\psi}_\mu \gamma^\mu \Gamma^j \phi_{ij} + \frac{1}{4} \phi^{ij} \mathcal{C}_{ij} \right). \quad (4.9)$$

5. Poincaré supergravity coupled to $N(2, 0)$ tensor multiplets from the superconformal theory

In this section we construct the matter couplings to $(2, 0)$ supergravity by using the superconformal tensor calculus and by imposing the $SO(N, 5)$ symmetry. We hereby closely follow the procedure of coupling $N = 4$, $d = 4$ vector multiplets as in [14]. This procedure was first introduced in [15] and has been applied to obtain matter couplings in four dimensions for $N = 1$ [16], $N = 2$ [17] and $N = 4$ [18]. The basic idea is that there is a close relation between matter-coupled Poincaré and conformal supergravity theories. Starting for (a slight generalization of) the matter-coupled conformal supergravity theory constructed in the previous section, we simply gauge fix the conformal scale and S -supersymmetry transformations to reproduce existing results on $D = 6$ matter-coupled Poincaré supergravity [3, 4]. For a review of this technique see, for example, [19].

We begin by introducing $(N + 5)$ copies of the $(2, 0)$ tensor multiplets with fields $B_{\mu\nu}^I, \psi^{iI}, L_I^{ij}$ where $I = 1, \dots, N + 5$ labels the vector representation of $SO(N, 5)$. We have denoted the scalars by L_I^{ij} because they will shortly be constrained. The constraint will be solved in terms of independent scalar fields which will again be denoted by ϕ .

The superconformal transformation rules now read

$$\begin{aligned} \delta B_{\mu\nu}^I &= -\bar{\epsilon} \gamma_{\mu\nu} \psi^I + \bar{\epsilon}^i \gamma_{[\mu} \psi_{\nu]}^j L_{ij}^I, \\ \delta \psi^{iI} &= \frac{1}{48} H_{abc}^{I+} \gamma^{abc} \epsilon^i + \frac{1}{4} \mathcal{D} L^{Iij} \epsilon_j - L^{Iij} \eta_j, \\ \delta L^{Iij} &= -4\bar{\epsilon}^i \psi^j L^I - \Omega^{ij} \bar{\epsilon} \psi^I. \end{aligned} \quad (5.1)$$

Since the tensor multiplet fields occur linearly in the full field equations (4.5), the latter generalize to the case of $N + 5$ tensor multiplets as

$$\begin{aligned} H_{abc}^{I-} - \frac{1}{2} L_{ij}^I T_{abc}^{ij} &= 0, \\ \mathcal{D} \psi_I^i - \frac{1}{15} L_I^{kl} \chi_{kl}^i - \frac{1}{12} T_{abc}^{ij} \gamma^{abc} \psi_{Ij} &= 0, \\ \mathcal{D}^a \mathcal{D}_a L_{ij}^I - \frac{1}{15} D_{ij}^{kl} L_{kl}^I + \frac{1}{3} H_{abc}^{I+} T_{ij}^{abc} + \frac{16}{15} \bar{\chi}_{ij}^k \psi_k^I &= 0. \end{aligned} \quad (5.2)$$

Note that the index I is a global $SO(N + 5)$ index and consequently the derivatives of L_{ij}^I and ψ_I^i occurring in (5.1) and (5.2) are as defined earlier for ϕ_{ij} and ψ without any new connection terms to rotate the index I .

To obtain the Poincaré supergravity coupled to N copies of the (2, 0) tensor multiplet, we impose the geometrical constraint

$$\eta^{IJ} L_I^{ij} L_{Jk\ell} = -\delta_{[k}^{[i} \delta_{\ell]}^{j]} + \frac{1}{4} \Omega^{ij} \Omega_{k\ell} \equiv \eta^{ij}_{k\ell} \quad (5.3)$$

where η_{IJ} is a symmetric invariant tensor of $SO(N, 5)$ with signature $(- - - - + + \dots +)$. The raising and lowering of the $SO(N, 5)$ indices will always be done with the metric η_{IJ} . The condition (5.3), together with the fact that L_I^{ij} are defined up to local $USp(4)$ transformations, reduces the number of independent scalars to $(N + 5) \times 5 - 15 - 10 = 5N$, which is the dimension of the coset $\frac{SO(N,5)}{SO(N) \times SO(5)}$. It is convenient to introduce an $(N + 5) \times N$ matrix L_I^r ($r = 1, \dots, N$) which, together with L_I^{ij} , form an $(N + 5) \times (N + 5)$ matrix L_I^A satisfying the condition

$$\eta^{IJ} L_I^A L_{JB} = \eta^A_B, \quad (5.4)$$

where $A = (ij, r)$ and the η^A_B is the constant metric with components: $\eta^{ij}_{k\ell} = \delta^r_s$ and $\eta^{ij}_r = \eta^r_{ij} = 0$.

The constraint (5.3) is invariant under S -supersymmetry. However, varying it under Q -supersymmetry gives the constraint

$$L_I^{ij} \psi_k^I = 0. \quad (5.5)$$

This constraint is easily solved as

$$\psi_I^i = L_I^r \psi^{ri}, \quad (5.6)$$

where ψ^{ri} ($r = 1, \dots, N$) are the independent fermionic fields.

Next, we vary the traceless part of equation (5.5) to obtain the constraint

$$L_I^{ij} \mathcal{D}_\mu L_{k\ell}^I = -8 \bar{\psi}^{I[i} \gamma_\mu \psi_{I[k} \delta_{\ell]}^{j]}, \quad (5.7)$$

and, making use of equation (2.9), the equation of motion for the 2-form potential is

$$H_{abc}^+ L_I^{ij} = -2 \bar{\psi}_I^i \gamma_{abc} \psi^{jI}. \quad (5.8)$$

Requiring that the trace part of the constraint (5.5) is invariant under the combined Q - and S -transformations, and using equations (5.6)–(5.8) and performing Fierz rearrangement, we determine the S -supersymmetry parameter:

$$\eta_i = -\frac{1}{2} (\bar{\psi}_i^r \gamma^a \psi_r^k) \gamma_{a\epsilon_k} - \frac{1}{72} (\bar{\psi}_i^r \gamma^{abc} \psi_r^k) \gamma_{abc\epsilon_k} - \frac{1}{36} (\bar{\psi}_r^k \gamma^{abc} \psi_k^r) \gamma_{abc\epsilon_i}. \quad (5.9)$$

Next, we observe that V_μ^{ij} can be solved from equation (5.7) as

$$V_{\mu i}^j = 2 L_{ik}^I \mathcal{D}'_\mu L_I^{jk} - 8 \bar{\psi}_I^I \gamma_\mu \psi_I^j, \quad (5.10)$$

where \mathcal{D}'_μ is the supercovariant derivative without the V_μ^{ij} term. The Weyl multiplet fields T_{abc}^{ij} , χ_{jk}^i and $D_{k\ell}^{ij}$ are also readily solved from equation (5.2). For example,

$$T_{abc}^{ij} = -2 H_{abc}^{I-} L_I^{ij}. \quad (5.11)$$

Using K -symmetry, we can also set

$$b_\mu = 0. \quad (5.12)$$

The independent fields we are left with are those of the combined (2, 0) Poincaré supergravity plus N tensor multiplet system, namely:

$$e_\mu^a, \quad \psi_\mu^i, \quad B_{\mu\nu}^I, \quad \psi_i^r, \quad L_I^A. \quad (5.13)$$

The Poincaré supersymmetry transformations of these fields can be found from equations (3.7) and (4.1) by using the solutions for the Weyl multiplet fields and the compensating S -supersymmetry transformation, equation (5.9). We thus find

$$\begin{aligned} \delta e_\mu^a &= \frac{1}{2} \bar{\epsilon} \gamma^a \psi_\mu, \\ \delta \psi_\mu^i &= \mathcal{D}_\mu \epsilon^i - \frac{1}{12} L_I^{ij} H_{\rho\sigma\tau}^{I-} \gamma^{\rho\sigma\tau} \gamma_\mu \epsilon_j + \gamma_\mu \eta^i, \\ \delta B_{\mu\nu}^I &= -L_r^I \bar{\epsilon}^i \gamma_{\mu\nu} \psi_i^r + L_{ij}^I \bar{\epsilon}^i \gamma_{[\mu} \psi_{\nu]}^j, \\ \delta \psi^{ir} &= \frac{1}{48} L_I^r H_{\mu\nu\rho}^{I+} \gamma^{\mu\nu\rho} \epsilon^i + \frac{1}{4} V_\alpha^{r,ij} \mathcal{D} \phi^\alpha \epsilon_j - \delta \phi^\alpha A_\alpha^{rs} \psi_s^i, \\ \delta \phi^\alpha &= 4 V_{r,ij}^\alpha \bar{\epsilon}^i \psi^{jr}, \end{aligned} \quad (5.14)$$

where ϕ^α ($\alpha = 1, \dots, 5N$) are the scalar fields parametrizing the coset $\frac{SO(N,5)}{SO(N) \times SO(5)}$ and η^i is given in equation (5.9). The vielbein $V_\alpha^{r,ij}$, the $SO(N)$ connection A_α^{rs} and the $USp(4)$ connection A_α^{ij} on this coset are defined as

$$\begin{aligned} V_\alpha^{r,ij} &= L^{Ir} \partial_\alpha L_I^{ij}, \\ A_\alpha^{rs} &= L^{Ir} \partial_\alpha L_I^s, \\ A_{\alpha i}^j &= 2 L_{ik}^I \partial_\alpha L_I^{jk}. \end{aligned} \quad (5.15)$$

Further definitions are as follows:

$$\begin{aligned} H_{\mu\nu\rho}^I &= 3 \partial_{[\mu} B_{\nu\rho]}^I + 3 \bar{\psi}_{[\mu} \gamma_{\nu\rho]} \psi^r L_r^I - \frac{3}{2} \bar{\psi}_{[\mu}^i \gamma_\nu \psi_{\rho]}^j L_{ij}^I, \\ \mathcal{D}_\mu \phi^\alpha &= \partial_\mu \phi^\alpha - 4 V_{r,ij}^\alpha \bar{\psi}_\mu^i \psi^{jr}, \\ \mathcal{D}_\mu \epsilon^i &= \partial_\mu \epsilon^i + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon^i - \frac{1}{2} V_\mu^i{}_j \epsilon^j, \end{aligned} \quad (5.16)$$

where the composite connection V_μ^{ij} is given by

$$V_\mu^{ij} = \mathcal{D}_\mu \phi^\alpha A_\alpha^{ij} - 8 \bar{\psi}_\mu^i \gamma_\mu \psi^{jr}. \quad (5.17)$$

Comparing the result (5.14) with that of [4], we find that all the structures are in agreement except the last term in $\delta \psi^{ir}$, which is missing in [4].

The self-duality condition (5.8) serves as the full field equation for the 2-form potential $B_{\mu\nu}^I$. The remaining field equations follow from the closure of the algebra (5.14). The resulting field equations can be found in [3, 4].

Summarizing, in this section we have shown that the $(2, 0)$ matter-coupled Poincaré theory of [3, 4] can be reproduced by fixing the conformal gauges in the $(2, 0)$ matter-coupled conformal supergravity constructed in this paper.

6. Conclusions

In this paper we have constructed the local conformal supersymmetry rules for $(2, 0)$ supergravity in six dimensions. That includes the transformation rules for the Weyl multiplet, equations (3.7), which is the gauge multiplet of the $OSp(8^*|4)$ superconformal algebra and the transformation laws (4.1) of the tensor multiplet. The latter has field equations given by (4.5). These results can be viewed as the quadratic approximation to the coupling of the full $M5$ -brane theory to conformal supergravity in a physical gauge. It would be interesting to obtain the full coupling of the $M5$ -brane to the $(2, 0)$ conformal supergravity.

Taking $N + 5$ copies of the tensor multiplets and imposing the constraints described in section 5 reproduces earlier results on Poincaré supergravity theory coupled to N tensor multiplets [3, 4]. The generalization of these results to the case of N coincident $M5$ -branes is, of course, a nontrivial problem.

We expect that the results obtained in this paper will have applications to the study of the AdS_7/CFT_6 correspondence. So far, very few results exist that deal with the calculation of

the correlation functions on the boundary of AdS_7 [20, 21]. Clearly, much remains to be done to develop a better understanding of this correspondence and the (2, 0) conformal supergravity ought to play a role in this process.

Another open problem of interest is the construction of the higher spin-operators of the (2, 0) tensor multiplets [22] and their coupling to appropriate higher spin-conformal supergravity fields. Of special interest are the operators which correspond to massless higher spin fields in the bulk of AdS_7 . These arise from the product of two doubleton representations of $OSp(8^*|4)$ [23]. It is natural that these operators couple to massless higher spin representation of this group. A field-theoretic realization of a higher spin AdS_7 supergravity is an interesting and challenging problem at present.

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Appendix A. Notations and conventions

We use the same notations as in [5], apart from the fact that we now use indices from 0 to 5 with signature $(- + \dots +)$ rather than the Pauli convention with indices from 1 to 6 with signature $(+ \dots +)$. Therefore the Levi-Civita tensor is adapted. We replace in [5]

$$i\epsilon_{abcdef} \rightarrow \epsilon_{abcdef}, \quad (A.1)$$

such that we now have

$$\epsilon_{012345} = 1 = -\epsilon^{012345}, \quad \gamma_7 = \gamma^0 \dots \gamma^5 = -\gamma_0 \dots \gamma_5. \quad (A.2)$$

The essential formula is as in [5]

$$\gamma_{abc}\gamma_7 = -\tilde{\gamma}_{abc}, \quad (A.3)$$

where the dual is now defined in (2.1).

We raise and lower $USp(4)$ indices with Ω^{ij} as:

$$\lambda^i = \Omega^{ij}\lambda_j, \quad \lambda_i = \lambda^j\Omega_{ji}. \quad (A.4)$$

When $USp(4)$ indices are omitted, northwest–southeast contraction is understood, e.g.

$$\bar{\lambda}\gamma^{(n)}\psi = \bar{\lambda}^i\gamma^{(n)}\psi_i, \quad (A.5)$$

where we have used the following notation:

$$\gamma^{(n)} = \gamma^{a_1 \dots a_n} = \gamma^{[a_1}\gamma^{a_2} \dots \gamma^{a_n]}. \quad (A.6)$$

The anti-symmetrizations are always with unit strength. Changing the order of spinors in a bilinear leads to the following signs:

$$\bar{\psi}^{(1)}\gamma^{(n)}\chi^{(2)} = t_n\bar{\chi}^{(2)}\gamma^{(n)}\psi^{(1)}, \quad \begin{cases} t_n = -1 & \text{for } n = 0, 3, 4 \\ t_n = 1 & \text{for } n = 1, 2, 5, 6 \end{cases} \quad (A.7)$$

where the labels (1) and (2) denote any $USp(4)$ representation, e.g. (1) = i and (2) = $[jk]$.

We frequently use the following Fierz rearrangement formula:

$$\psi_j \bar{\psi}^i = -\frac{1}{4}(\bar{\psi}^i \gamma_a \psi_j) \gamma^a + \frac{1}{48}(\bar{\psi}^i \gamma_{abc} \psi_j) \gamma^{abc}. \quad (\text{A.8})$$

The notation ‘-(trace)’ denotes terms that are proportional to either Ω^{ij} or δ_j^i (with ‘free’ indices). We use the notation ‘-(traces)’ if both invariant tensors occur. For the convenience of the reader we give below the explicit expressions of some trace terms:

$$\begin{aligned} X^{ij} - (\text{trace}) &= X^{ij} + \frac{1}{4} \Omega^{ij} X^k_k, \\ A^{ij} X_k - (\text{traces}) &= A^{ij} X_k + \frac{4}{5} A^{\ell[i} X_\ell \delta_k^{j]} - \frac{1}{5} \Omega^{ij} A_{k\ell} X^\ell, \\ S_k^{[i} X^{j]} - (\text{traces}) &= S_k^{[i} X^{j]} - \frac{1}{5} \delta_k^{[i} S^{j]\ell} X_\ell + \frac{1}{5} \Omega^{ij} S_k^\ell X_\ell, \end{aligned} \quad (\text{A.9})$$

where X^i and X^{ij} are arbitrary $USp(4)$ tensors, while A^{ij} is an antisymmetric traceless and S^{ij} a symmetric tensor.

Appendix B. The $(2, 0) \rightarrow (1, 0)$ truncation

Many of the formulae for the $(2, 0)$ Weyl and tensor multiplet can be obtained by considering their truncations to the $(1, 0)$ case and comparing with the results of [5]. Following [24], the $(2, 0)$ Weyl multiplet may also be compared with the $N = 4, D = 4$ Weyl multiplet of [18].

We first consider the $(2, 0)$ Weyl multiplet. The $(2, 0)$ Weyl multiplet (3.7) leads to the $N = 2$ Weyl multiplet of [5] (see equation (2.26)) upon making the following truncations. We write $i = 1, \dots, 4 = (i = 1, 2, i' = 1, 2)$, and we put

$$\Omega^{ij} = \begin{pmatrix} \epsilon^{ij} & 0 \\ 0 & \epsilon^{i'j'} \end{pmatrix}. \quad (\text{B.1})$$

The non-vanishing bosonic component fields are given by

$$\begin{aligned} V_\mu^{ij} &= V_\mu^{ij}, \\ T_{abc}^{ij} &= \epsilon^{ij} T_{abc}, & T_{abc}^{i'j'} &= -\epsilon^{i'j'} T_{abc}, \\ D_{kl}^{ij} &= \epsilon^{ij} \epsilon_{kl} D, & D_{kl}^{ij} &= -\epsilon^{ij} \epsilon_{k'l'} D, & D_{kl}^{i'j'} &= -\epsilon^{i'j'} \epsilon_{kl} D, \\ D_{kl}^{i'j'} &= \epsilon^{i'j'} \epsilon_{k'l'} D, & D_{kl}^{ij} &= -\frac{1}{2} \delta_k^i \delta_l^{j'} D. \end{aligned} \quad (\text{B.2})$$

For example, the first equation above means that $V_\mu^{ij'} = 0 = V_\mu^{i'j'}$. The non-vanishing fermionic component fields are given by

$$\begin{aligned} \psi_\mu^i &= \psi_\mu^i, \\ \chi_{ij}^k &= \epsilon_{ij} \chi^k, & \chi_{i'j'}^k &= -\epsilon_{i'j'} \chi^k, & \chi_{i'j'}^{k'} &= -\frac{1}{2} \delta_{i'}^{k'} \chi_j. \end{aligned} \quad (\text{B.3})$$

Thus, for example, $\psi^{i'} = 0$. Finally, the non-vanishing supersymmetry parameters are given by

$$\epsilon^i = \epsilon^i, \quad \eta^i = \eta^i, \quad (\text{B.4})$$

which means that $\epsilon^{i'} = 0 = \eta^{i'}$. In comparing the truncated result with the $(1, 0)$ Weyl multiplet of [5], two remarks are in order. First of all, the $(1, 0)$ conventional constraints of [5] contain extra χ - and D -dependent terms which do not generalize to the $(2, 0)$ case. As a consequence the dependent K and S gauge fields, obtained after truncation, differ from those of [5]. In order to obtain the truncated result one should replace the K and S gauge fields of [5] by the following expressions:

$$\begin{aligned} f_\mu^a &\rightarrow f_\mu^a + \frac{1}{240} e_\mu^a D, \\ \phi_\mu^i &\rightarrow \phi_\mu^i - \frac{1}{60} \gamma_\mu \chi^i. \end{aligned} \quad (\text{B.5})$$

Secondly, in order to remove the χ -dependent term from the supersymmetry variation of the dilatation gauge field b_μ (again, this term cannot be generalized to the (2, 0) case) one must perform a field-dependent K -transformation on the results of [5] with the following parameter:

$$\lambda_{K\mu} = -\frac{1}{60}\bar{\epsilon}\gamma_\mu\chi. \quad (\text{B.6})$$

The net effect of these manipulations is that all χ -dependent terms in the (1, 0) theory that cannot be extended to the (2, 0) case are being removed.

Next we consider the (2, 0) tensor multiplet. The truncation of this multiplet to the (1, 0) case treated in [5] is given by:

$$\begin{aligned} \phi^{ij} &= \epsilon^{ij}\sigma, & \phi^{i'j'} &= -\epsilon^{i'j'}\sigma, & \phi^{ij'} &= 0, \\ \psi^i &= \psi^i, & \psi^{i'} &= 0. \end{aligned} \quad (\text{B.7})$$

In order to show that the ϕ^{ij} field equation (see the third equation of (4.5)) truncates correctly to the (1, 0) equation (see equation (3.27) of [5]) one has to take special care of the ϕD , $\bar{\psi}_\mu\chi\phi$ and $\bar{\chi}\psi$ terms. Concerning the $D\phi$ term, starting from the (1, 0) case, the redefinition equation (B.5) of f_a^a leads to an extra $D\sigma$ term which is added to the explicit $D\sigma$ term in the equation of motion (3.27) of [5]. As for the $\bar{\psi}_\mu\chi\phi$ terms, the redefinition of ϕ_μ (see equation (B.5)) in the $\bar{\psi}_\mu D\psi$ term plus the compensating K -transformation given in (B.6) lead to two extra $\bar{\psi}_\mu\chi\phi$ terms such that the total contribution cancels. This is consistent with the fact that the truncation of the $\bar{\psi}_\mu\chi\phi$ term in equation (4.8) vanishes identically. Finally, the redefinition of ϕ^μ in the $\bar{\psi}\phi_\mu$ term in $\mathcal{D}^a\mathcal{D}_a$ (see equation (3.30) of [5]) leads to an extra $\bar{\chi}\psi$ term which should be added to the explicit such contribution in the σ field equation. The total then agrees with the (1, 0) truncation of our result, equation (4.5).

Appendix C. Tensor, current and Weyl multiplets in superspace

The (2, 0) tensor multiplet in flat superspace can be described by a superfield ϕ^{ij} satisfying the constraint [9]

$$D_\alpha^i\phi^{jk} = \Omega^{[ij}\lambda_\alpha^{k]} - (\text{trace}). \quad (\text{C.1})$$

In flat superspace the current multiplet of equation (2.7) is described by the supercurrent [9]

$$J^{ij,kl} = \phi^{ij}\phi^{kl} - (\text{trace}), \quad (\text{C.2})$$

where the superfield ϕ^{ij} describes the (2, 0) tensor multiplet.

In superspace the Weyl multiplet (3.7) is described by an anti-self-dual superfield W_{abc}^{ij} in the **5** of $USp(4)$, whose first component is the bosonic field T_{abc}^{ij} and which satisfies the constraint

$$D_{\alpha i}W_{abc}^{jk} = \delta_i^{[j}(\gamma^{de}\gamma_{abc}\chi_{de}^{k]})_\alpha + (\gamma_{abc}\lambda_i^{jk})_\alpha - (\text{trace}), \quad (\text{C.3})$$

where λ_i^{jk} is in the **16** of $USp(4)$.

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